

**HARD THERMAL LOOPS, QUARK-GLUON PLASMA RESPONSE
AND $T = 0$ TOPOLOGY¹**

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E-mail: lucchesi@iph.unine.ch***Abstract**

I outline various derivations of the non-Abelian Kubo equation, which governs the response of a quark-gluon plasma to hard thermal perturbations. In the static case, it is proven that gauge theories do not support hard thermal solitons. Explicit solutions are constructed within an $SU(2)$ Ansatz and they are shown to support the general result. The time-dependent problem, *i.e.*, non-Abelian plasma waves, has not been completely solved. We express and motivate the hope that the intimate relations linking the gauge-invariance condition for hard thermal loops to the equation of motion for $T = 0$, topological Chern-Simons theory may yield new insight into this field.

1. The Quark-Gluon Plasma

The study of the quark-gluon plasma (QGP) phase of high-temperature QCD₄ is relevant both to big-bang cosmology and to heavy ions collisions. Experiments at CERN and Brookhaven have provided compelling evidence for the existence of an intermediate “phase” of QCD during such collisions, with an energy density $\sim 1 \text{ GeV}/fm^3$ (see for instance [1]). It is however not clear if this “phase” is a thermodynamically distinct one.

A heavy-ions interaction, after the primary collision of ions bunches, forms a thermalized state which lasts for most of the total interaction time. It is in this thermalized state that the QGP “phase” is thought to be formed. The interaction ends by hadronization of the thermalized constituents. Each step of the process can be tested specifically [1, 2]. However, the methods

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available for analysis are empirical, and one feels compelled to work towards developing a more satisfactory theoretical framework for the investigation of the QGP.

2. High-Temperature Perturbative QCD

High-temperature perturbative QCD provides a natural setting for the theoretical study of the QGP. We shall adopt the hypotheses that (i) QCD is in its deconfined phase and that (ii) the plasma is collisionless, so that it can be regarded as a dilute gas of quarks and gluons. The dominant interaction is between hard colored plasma particles and the soft mean color field. “Hard” denotes an energy scale of the order of the temperature, $\varepsilon \sim T$, while “soft” refers to the energy scale $\varepsilon \sim gT$, g being the gauge coupling constant. It is to expect that motion on a distance scale $\sim 1/gT$, the inverse soft energy scale, should involve coherently many hard particles, giving rise to color polarization effects in the QGP. Indeed, the real part of the color polarization tensor describes Debye screening of the color charges, while its imaginary part accounts for Landau damping of the mean color field.

If zero-temperature perturbative QCD is naively extended to the $T > 0$ case, physical quantities turn out to be gauge-dependent. This old puzzle has been solved by the discovery of the so-called “hard thermal loops” (HTLs) phenomenon [3]: at high temperature, effects of leading order in g can arise at any order of perturbation theory, *i.e.*, loop diagrams exist that are comparable to the corresponding tree amplitude. Such diagrams contain one-loop integrals with soft external lines and hard internal (loop) momenta, and are termed “hard thermal loops”. In order to account consistently for the orders in g , one is led to resum the perturbative expansion.

Hard thermal loops represent thermal, not quantum, corrections. They describe polarization effects in the plasma, as well as the propagation of plasma waves, plasma solitons, etc. The hard thermal loops are the dominant diagrams at high temperature. They are proportional to the Debye mass squared $m_D^2 = \frac{g^2 T^2}{3} (N + \frac{N_F}{2})$, where N is the number of colors and N_F the number of flavors. Note that the Debye mass is proportional to the soft scale gT ; m_D represents the smallest possible frequency for time-dependent (gluon) propagation. The Debye screening length is just its inverse, $\lambda_D = m_D^{-1} \sim 1/gT$.

3. The Non-Abelian Kubo Equation

Hard thermal loops are generated by an effective action [3, 4]:

$$\Gamma_{\text{HTL}}(A) = \frac{m_D^2}{2} \left[\int d^4x A_0^a A_0^a + \int \frac{d\Omega_{\hat{\mathbf{q}}}}{2\pi} W(A_+) \right]. \quad (3.1)$$

The first term in the right side describes color-electric screening, and $W(A_+)$ is a non-local functional of $A_+ \equiv \frac{1}{\sqrt{2}}(1; \hat{\mathbf{q}}) \cdot A$, where $\hat{\mathbf{q}}$ is an arbitrary unit 3-vector, $\hat{\mathbf{q}}^2 = 1$. The measure $d\Omega_{\hat{\mathbf{q}}}/2\pi$ denotes integration over the solid angle spanned by $\hat{\mathbf{q}}$.

The response of the QGP to an external perturbation is governed by a Yang-Mills field

equation

$$D_\mu F^{\mu\nu} = \frac{m_D^2}{2} J_{\text{induced}}^\mu, \quad (3.2)$$

with the induced hard thermal current $\frac{m_D^2}{2} J_{\text{induced}}^\mu \equiv -\frac{\delta}{\delta A_\mu} \Gamma_{\text{HTL}}(A)$ as its source¹. In order to close the system of differential equations for A_μ , we need a constraint on the current. Exploiting the analogy between the gauge invariance of HTLs and Chern-Simons theory² [5], one derives the induced current in the form [6]

$$J_{\text{induced}}^\mu = \int \frac{d\Omega_{\hat{\mathbf{q}}}}{4\pi} \left(Q_+^\mu [a_-(x) - A_-(x)] + Q_-^\mu [a_+(x) - A_+(x)] \right), \quad (3.3)$$

where $Q_\pm^\mu \equiv \frac{1}{\sqrt{2}}(1, \pm \hat{\mathbf{q}})$, $\hat{\mathbf{q}}^2 = 1$, are light-like 4-vectors, $A_\pm = Q_\pm^\mu A_\mu$ are light-like projections of A_μ , and a_\pm are constrained, introducing $\partial_\pm = Q_\pm^\mu \partial_\mu$, by

$$\partial_+ a_- - \partial_- A_+ + [A_+, a_-] = 0, \quad \partial_+ A_- - \partial_- a_+ - [A_-, a_+] = 0. \quad (3.4)$$

From now on, we shall call *non-Abelian Kubo equation* the system (3.2)–(3.4), which governs hard thermal processes in the QGP.

Alternative derivations of the non-Abelian Kubo equations have been given.

1. The non-Abelian Kubo equation can be derived from quantum kinetic theory [7]. In that approach, one starts from the Schwinger-Dyson equations, and uses a kinematic approximation scheme in which derivatives with respect to center-of-mass and to relative coordinates carry different g -dependences. From the resulting quantum kinetic equations, one is able to derive $\Gamma_{\text{HTL}}(A)$, and hence an expression for the induced current.

2. The non-Abelian Kubo equation can also be derived, as done in [8], by applying the action principle to the Cornwall-Jackiw-Tomboulis composite effective action $\Gamma[\phi, G_\phi]$ [9], a functional of a generic field ϕ (which stands for gluons, quarks, antiquarks, ghosts, etc) and of the two-point functions G_ϕ . Requiring that $\Gamma[\phi, G_\phi]$ be stationary with respect to $\phi(x)$ and $G_\phi(x, y)$ yields, respectively, the equation (3.2) and a set of conditions on the two-point functions which can be cast in the form of a constraint on the induced current [8].

This approach is close to the one of [7], of which it adopts numerous steps, including its kinematic approximation scheme. The composite action approach presents the advantage of providing an action principle for hard thermal processes in the quark-gluon plasma. Whether, and which, physical hypotheses can be expressed at the level of the composite action functional is an open question.

3. More recently, the non-Abelian Kubo equation has been derived from classical transport theory [10]. This is the subject of the talk by C. Manuel at this meeting. One starts from classical particle dynamics as described by the Wong equations; the color charge is considered as

¹Note that (3.2) is just the equation of motion for the gauge field, when one adds the zero-temperature QCD action to the HTLs generating functional, *i.e.*, (3.2) is equivalent to requiring $\frac{\delta}{\delta A_\mu} (S_{\text{QCD}}^{T=0} + \Gamma_{\text{HTL}}) = 0$.

²See Section 5 below for a more detailed account of this analogy.

a *classical* phase-space variable. Transport theory yields the non-Abelian Vlasov equations, and the induced current is expressed in terms of the one-particle distribution functions. Expanding these distribution functions in powers of the coupling constant g yields, at leading order, the gauge invariance condition for the generating functional of hard thermal loops $\Gamma_{\text{HTL}}(A)$ (3.1). Subsequent developments that lead to the form (3.3)–(3.4) for the induced current, and hence to the non-Abelian Kubo equation, are described in [6].

4. Solutions of the Non-Abelian Kubo Equation: QGP Response

It is of obvious interest to discuss solutions of the non-Abelian Kubo equation (3.2)–(3.4). In the Abelian, electrodynamical case this is easy to do, since the conditions (3.4) can be readily solved for a_{\pm} , and the solutions of the linear problem are the well-known Abelian plasma waves [11]. The non-linear problem of finding non-Abelian plasma waves is much more formidable. Also, one inquires whether the non-linear equations support soliton (and instanton) solutions.

Let us first consider the static case. Acting on time-independent fields $A_{\pm} = A_{\pm}(\mathbf{q})$, the derivatives in (3.4) become $\partial_{\pm} = \pm \frac{1}{\sqrt{2}} \hat{\mathbf{q}} \cdot \nabla \equiv \pm \partial_{\mathbf{q}}$; therefore the constraints (3.4) can be written as

$$\partial_{\mathbf{q}} (A_+ + a_-) + [A_+, A_+ + a_-] = 0, \quad \partial_{\mathbf{q}} (A_- + a_+) - [A_-, A_- + a_+] = 0. \quad (4.1)$$

These constraints are solved by $a_- = -A_+$, resp. $a_+ = -A_-$. (3.3) then yields

$$J_{\text{induced}}^{\mu}(\mathbf{q}) = - \int \frac{d\Omega_{\hat{\mathbf{q}}}}{4\pi} (Q_+^{\mu} + Q_-^{\mu}) (Q_+^{\nu} + Q_-^{\nu}) A_{\nu}(\mathbf{q}) \quad (4.2)$$

for the static inducent current. With $\mathbf{Q}_+ + \mathbf{Q}_- = 0$ and $Q_+^0 + Q_-^0 = \sqrt{2}$, one computes $J_{\text{induced}}^{\mu} = (-2 A^0; \mathbf{0})$. The response equations (3.2) become, in the static limit:

$$D_i E^i = -m_D^2 A_0, \quad \epsilon^{ijk} D_j B^k - [A_0, E^i] = 0, \quad (4.3)$$

where $E^i \equiv F^{i0}$ and $F^{ij} \equiv -\epsilon^{ijk} B^k$, and the Debye mass m_D plays the role of a gauge invariant, color-electric screening mass.

In the study of static excitations, one is of course interested in the issue of possible (static) soliton solutions. It has been established in [8] that the non-Abelian Kubo equation does not possess any finite energy static solutions. The proof goes as follows. Consider the symmetric tensor

$$\theta^{ij} = 2 \text{Tr} \left(E^i E^j + B^i B^j - \frac{\delta^{ij}}{2} (E^2 + B^2 + m_D^2 A_0^2) \right). \quad (4.4)$$

Using (4.3) one verifies that for static fields $\partial_j \theta^{ji} = 0$. Therefore

$$\int d^3r \theta^{ii} = \int d^3r \partial_j (x^i \theta^{ji}) = \int dS^j x^i \theta^{ji}. \quad (4.5)$$

Moreover, the energy of a massive gauge field (with no mass for the spatial components) can be written as

$$\mathcal{E} = \int d^3r \left\{ -\text{Tr} \left(E^2 + B^2 + \frac{1}{m_D^2} (D_i E^i)^2 \right) + \text{Tr} \left(m_D A_0 + \frac{D_i E^i}{m_D} \right)^2 \right\}. \quad (4.6)$$

The second trace in the integrand enforces the first of constraints (4.3). Consequently, on the constrained surface the energy is a sum of positive terms,

$$\mathcal{E} = \int d^3r \left\{ -\text{Tr} \left(E^2 + B^2 + m_D^2 A_0^2 \right) \right\} , \quad (4.7)$$

and \mathbf{E} , \mathbf{B} and A_0 must decrease at large distances sufficiently rapidly so that each of them is square integrable. This in turn ensures that the surface integral at infinity in (4.5) vanishes, so that static solutions require $\int d^3r \theta^{ii} = 0$. On the other hand, from (4.4), we see that θ^{ii} is a sum of positive terms, $\theta^{ii} = -\text{Tr} (E^2 + B^2 + 3m_D^2 A_0^2)$, hence we must have $\mathbf{E} = \mathbf{B} = \mathbf{0}$ and $A_0 = 0$, *i.e.*, there are no (non-trivial) finite-energy solutions to (4.3). A similar argument [8] shows the absence of “static” instantons.

I want to analyze now in some details possible solutions of the static response equations, and present numerical results [8] that support the general proof of absence of (static) solitons. I consider for simplicity the radially symmetric restriction of the static response equations (4.3), in the $SU(2)$ case. Radially symmetric $SU(2)$ gauge potentials have the form³

$$A_0^a = \hat{r}^a \frac{g(r)}{r} , \quad A_i^a = (\delta^{ai} - \hat{r}^a \hat{r}^i) \frac{\phi_2(r)}{r} + \varepsilon^{aij} \hat{r}^j \frac{1 - \phi_1(r)}{r} . \quad (4.8)$$

We substitute this *Ansatz* into (4.3). The resulting equations give us the freedom to set ϕ_2 to zero. Rescaling $x = mr$ and defining $J(x) = g(r)$, $K(x) = \phi_1(r)$, we obtain a system of coupled second-order differential equations

$$x^2 \frac{d^2}{dx^2} J = (x^2 + 2K^2) J , \quad x^2 \frac{d^2}{dx^2} K = (K^2 - J^2 - 1) K . \quad (4.9)$$

These equations possess two exact solutions:

- (1) the *Yang-Mills vacuum*, with $J = 0$, $K = \pm 1$, and
- (2) the *Wu-Yang monopole*, plus a screened electric field: $J = J_0 e^{-x}$, $K = 0$.

Let us investigate the asymptotic behaviour of the system (4.9). At $x \rightarrow \infty$, the regular solution tends to the Yang-Mills vacuum, with J approaching its asymptote exponentially⁴. Near the origin, *i.e.*, at $x \rightarrow 0$, J and K behave either like the Yang-Mills vacuum, or approach the Wu-Yang monopole, as

$$J(x) \rightarrow J_0 + \dots , \quad K(x) \rightarrow K_0 \sqrt{x} \cos \left(\sqrt{J_0^2 + \frac{3}{4}} \ln \frac{x}{x_0} \right) + \dots . \quad (4.10)$$

Only the vacuum alternative at the origin could lead to finite energy. However, since we must choose one of two possible solutions at infinity (obviously we pick the regular one), the behavior at the origin is determined and can be exhibited explicitly by integrating the equations (4.9) numerically. Starting with regular boundary conditions at infinity, we find that the monopole solution is reached at the origin, with K vanishing as in (4.10); see Figure 1

³A residual gauge freedom has been used to eliminate a term proportional to $\hat{r}^a \hat{r}^i$.

⁴There is also a solution with J growing exponentially, which we do not consider.

below. This result is consistent with the analytic proof [8] that there are no finite energy static solutions in hard thermal gauge theories.

Much less is known about time-dependent solutions of the non-Abelian Kubo equation (3.2)–(3.4). The special case of time-dependent, space-independent solutions has been investigated in [12]. The *Ansatz* $A^\mu(x) = (0, \mathbf{A}(t))$ leads to the induced current $J_{\text{induced}}^\mu = (0; -\frac{2}{3} \mathbf{A})$, which describes global color oscillations in the plasma. A more general *Ansatz*, $A_\mu(x) = A_\mu(\mathbf{p} \cdot \mathbf{x} - \omega t)$, has been presented in [13] to describe non-Abelian plane waves in the plasma.

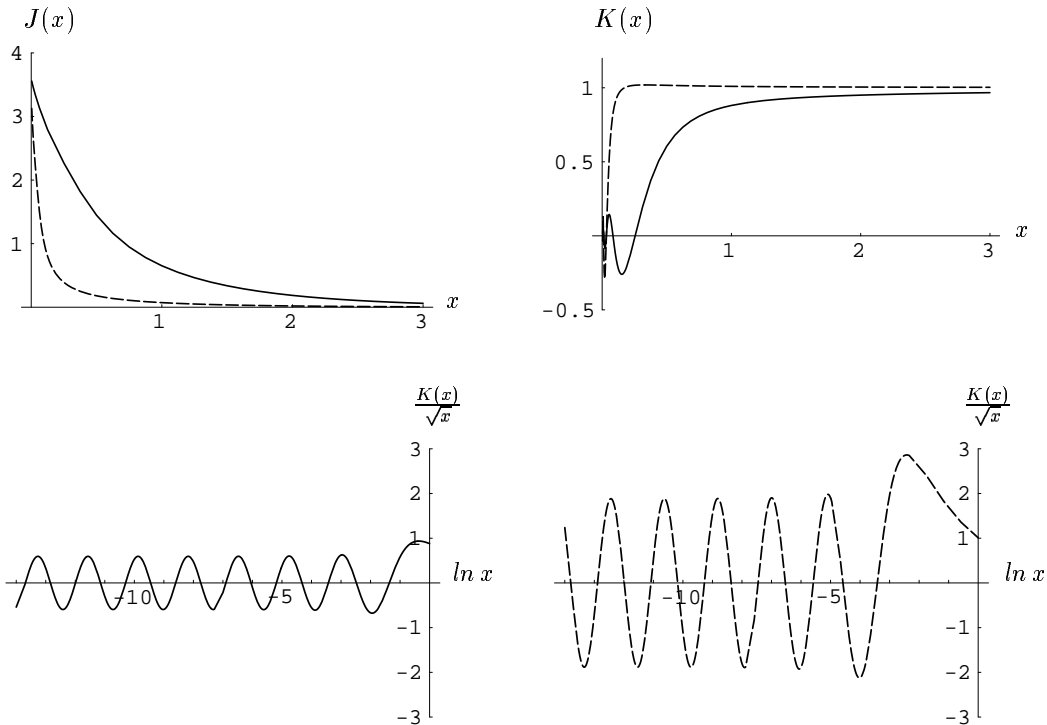


Figure 1: $J(x)$ and $K(x)$ obtained by integrating numerically equations (4.9), starting with regular boundary conditions at infinity. The plain and dashed lines represent different rates of approach to the (Yang-Mills vacuum) asymptotes at $x = \infty$, with $K = 1$. Analysis of the oscillations in K near the origin [see the plots of $K(x)/\sqrt{x}$ vs. $\ln(x)$] verifies the analytical form of the asymptotes (4.10).

5. QGP Response and $T = 0$ Topology

In order to solve completely the time-dependent non-Abelian Kubo equation, one may need new insight, a possible source of which is provided by zero-temperature, topological gauge theory. Indeed, the gauge invariance condition for the generating functional of hard thermal loops in QCD₄ can be identified with the equation of motion for the gauge field in $T = 0$, $d = 3$ topological Chern-Simons theory [5].

Let us see how this works in some details. We first perform a gauge transformation $\delta_g A_\mu = \partial_\mu \omega + [A_\mu, \omega]$, with parameter $\omega = -it^a \omega^a$, of the HTLs generating functional (3.1).

Imposing gauge invariance of (3.1), $\delta_g \Gamma_{\text{HTL}}(A) = 0$, implies

$$\int d\Omega_{\hat{\mathbf{q}}} \delta_g W(A_+) = 4\pi \int d^4x (\partial_0 A_0^a) \omega^a. \quad (5.1)$$

Expressing $\delta_g W(A_+)$, upon integrating by parts, as

$$\delta_g W(A_+) = - \int d^4x \frac{\delta W(A_+)}{\delta A_+} \delta_g A_+ = - \int d^4x \left\{ \partial_+ \frac{\delta W}{\delta A_+} + \left[A_+, \frac{\delta W}{\delta A_+} \right] \right\}^a \omega^a$$

leads, upon introducing the combination $f \equiv \frac{\delta W}{\delta A_+} + A_+$, to the gauge invariance condition for hard thermal loops [4], written now as:

$$\frac{\partial f}{\partial u} + [A_+, f] + \frac{\partial A_+}{\partial v} = 0, \quad (5.2)$$

in the new space-time coordinates (u, v, \mathbf{x}^T) defined by $u \equiv Q_-^\mu x_\mu$, $v \equiv Q_+^\mu x_\mu$, and $\mathbf{Q}_+ \cdot \mathbf{x}^T = 0$.

To establish the relation with a topological action, one proceeds following [5]. First, perform a Wick rotation into Euclidean \mathbb{R}^4 space-time, and rename the coordinates $u = z$, $v = \bar{z}$. Next, introduce the notation $A_+ = a_z$ and define $a_{\bar{z}} \equiv -f$. Last, enforce the axial gauge condition $a_0 = 0$. The gauge invariance condition for HTLs (5.2) can then be rewritten as

$$\partial_{\bar{z}} a_z - \partial_z a_{\bar{z}} + [a_{\bar{z}}, a_z] = 0. \quad (5.3)$$

This is the zero-curvature condition $F_{z\bar{z}} = 0$ for the gauge field strength tensor $F_{z\bar{z}}$. I now switch to the Landau gauge, $\partial_\mu A^\mu = 0$, for purposes of illustration. In that gauge, the zero-curvature condition (5.3) takes the form $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu] = 0$. This is just the equation of motion for the gauge field A_μ with the classical action

$$S = \int d^3x \varepsilon^{\mu\nu\rho} \left(A_\mu^a \partial_\nu A_\rho^a + \frac{2}{3} A_\mu^a A_\nu^b A_\rho^c f^{abc} \right). \quad (5.4)$$

The latter action is the one of topological Chern-Simons theory, a zero-temperature gauge field theory in three space-time dimensions. “Topological” refers to the fact that the Lorentz indices in (5.4) are saturated without using the metric tensor $g_{\mu\nu}$. As a consequence, the energy-momentum tensor, computed as $T^{\mu\nu} = \frac{\delta S}{\delta g_{\mu\nu}}$, doesn’t get any contributions from the action (5.4). Only the (non-topological, *i.e.*, metric-dependent) gauge fixing and ghost terms that one adds to the action in order to quantize the theory contribute to $T^{\mu\nu}$, but such contributions do not represent physical energy-momentum.

Chern-Simons theory, together with other topological field theories, has been intensively studied in recent years [14], both due to its mathematical and physical interests. Chern-Simons theory describes anyons (particles with non-integer spins) and is a starting point for modeling the quantum Hall effect and high- T_c superconductivity. Chern-Simons theory is known to possess the appealing property of perturbative finiteness [15], which in turn is connected to an interesting (twisted) form of supersymmetry [16, 15] that is peculiar to topological theories.

The question whether such properties could be of interest in the realm of high- T QCD₄ is an open, and interesting one. In particular, the twisted topological supersymmetry of Chern-Simons theory [17] could reveal a useful tool to shed new light on the relationship between high temperature in field theory and (untwisted) physical supersymmetry.

6. Conclusions

There are indications that the quark-gluon plasma may have been observed experimentally in heavy-ions collisions. However, a satisfactory theory of the QGP phase is still lacking. The best available theoretical framework is high-temperature QCD, in which hard thermal effects are dominant.

The response of a quark-gluon plasma to a hard thermal perturbations is described by the non-Abelian Kubo equation. The static case is completely solved. In particular, there do not exist hard thermal solitons. This general result is confirmed by numerical computations, within a radial $SU(2)$ *Ansatz*. The time-dependent case is less well understood.

New insight into this field may hence be needed, and could indeed be provided by the close relationship that exists between hard thermal QCD₄ and three-dimensional Chern-Simons theory, a topological gauge theory at zero temperature. Chern-Simons theory is perturbatively finite. Furthermore, its gauge-fixed action exhibits a twisted supersymmetry which relates “bosons” (fields of even ghost number) to “fermions” (fields of odd ghost number), in very close analogy to usual (untwisted) supersymmetry. This leads us to the following, open questions. (1) Can the twisted supersymmetry of Chern-Simons theory be translated into the language of high-temperature QCD, and what does it mean in that context? (2) Can this shed new light onto the interplay between supersymmetry and high-temperature in field theory?

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